

## Mixed-Integer Optimization in GAMS using CAPE-OPEN Thermodynamics via MOSAICmodeling and COBIA for Conceptual Design

CAPE-OPEN 2019 Annual Meeting | 22.10.2019

David Krone, Erik Esche, Norbert Asprion, Jens-Uwe Repke, Mirko Skiborowski

---

# Motivation

## Optimization with thermodynamic calculations

### 1. Formulate all thermo functions explicitly

$$\begin{aligned} \min \quad & f(x) \\ s.t. \quad & g(x) = 0 \end{aligned}$$

→ Inflated model

- Complex initialization necessary
- No easy changes in thermo calculations (e.g. change from ideal to non-ideal liquid phase)



### 2. Function calls to thermo property packages

$$\begin{aligned} \min \quad & f(x, y) \\ s.t. \quad & g(x, y) = 0 \\ & y = \text{thermo}(x) \end{aligned}$$

→ Lean models with thermo flexibility



**Problem:** optimization tools do not usually provide integrated thermo packages

## Motivation

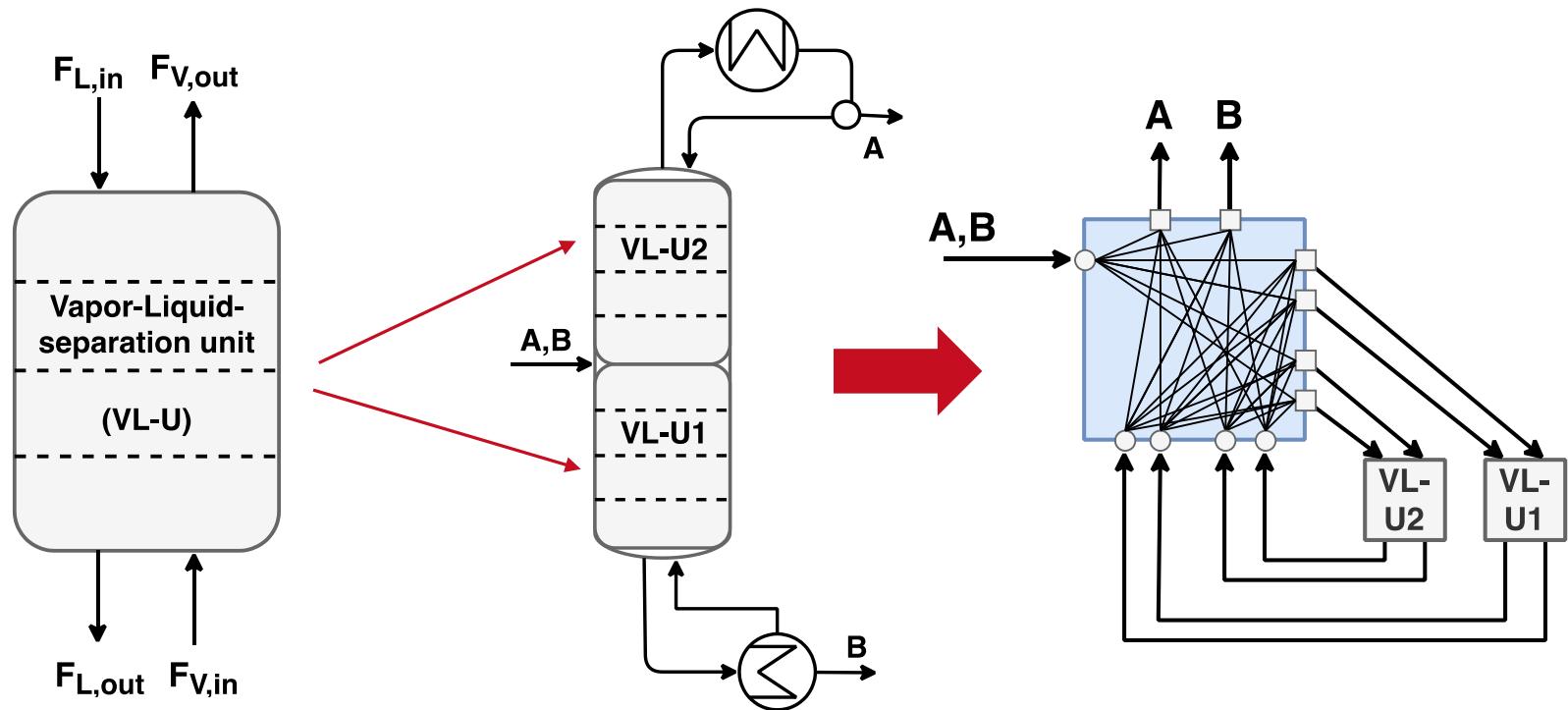
## Development Goal

Generate the **interface between**  
**GAMS and CAPE-OPEN thermo property packages**  
**for fast thermo calculations**

- 3 Mixed-Integer Optimization in GAMS using CO Thermodynamics via MOSAICmodeling and COBIA for Conceptual Design

D. Krone | CAPE-OPEN 2019 Annual Meeting | 22.10.2019

# I. Mixed-Integer(MI) Optimization for Conceptual Design



- 4 Mixed-Integer Optimization in GAMS using CO Thermodynamics via MOSAIcModeling and COBIA for Conceptual Design

D. Krone | CAPE-OPEN 2019 Annual Meeting | 22.10.2019

# I. MI Optimization for Conceptual Design

## The necessity of calculation speed

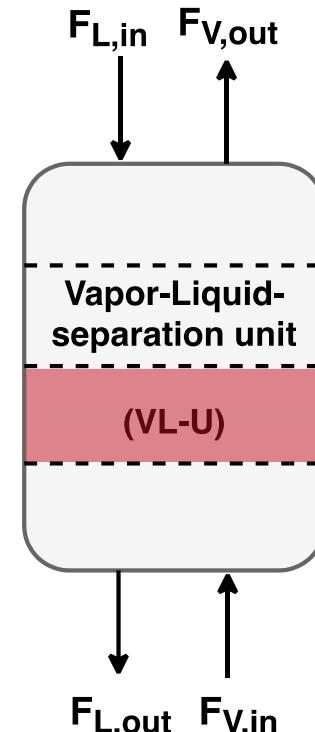
Number of thermo calls per equilibrium stage (ternary mixture)

property	Calls in GAMS program file	Calls to thermo package (C++)
single phase enthalpies ( $h^L$ and $h^V$ )	2	8
distribution coefficients $K_c$	3	18

→ 26 calls/stage

For four equilibrium stages per unit:

→ 104 calls/separation unit



- 5 Mixed-Integer Optimization in GAMS using CO Thermodynamics via MOSAICmodeling and COBIA for Conceptual Design

# I. Optimization-based Conceptual Design

## The necessity of calculation speed

→ 624 calls/all separation units

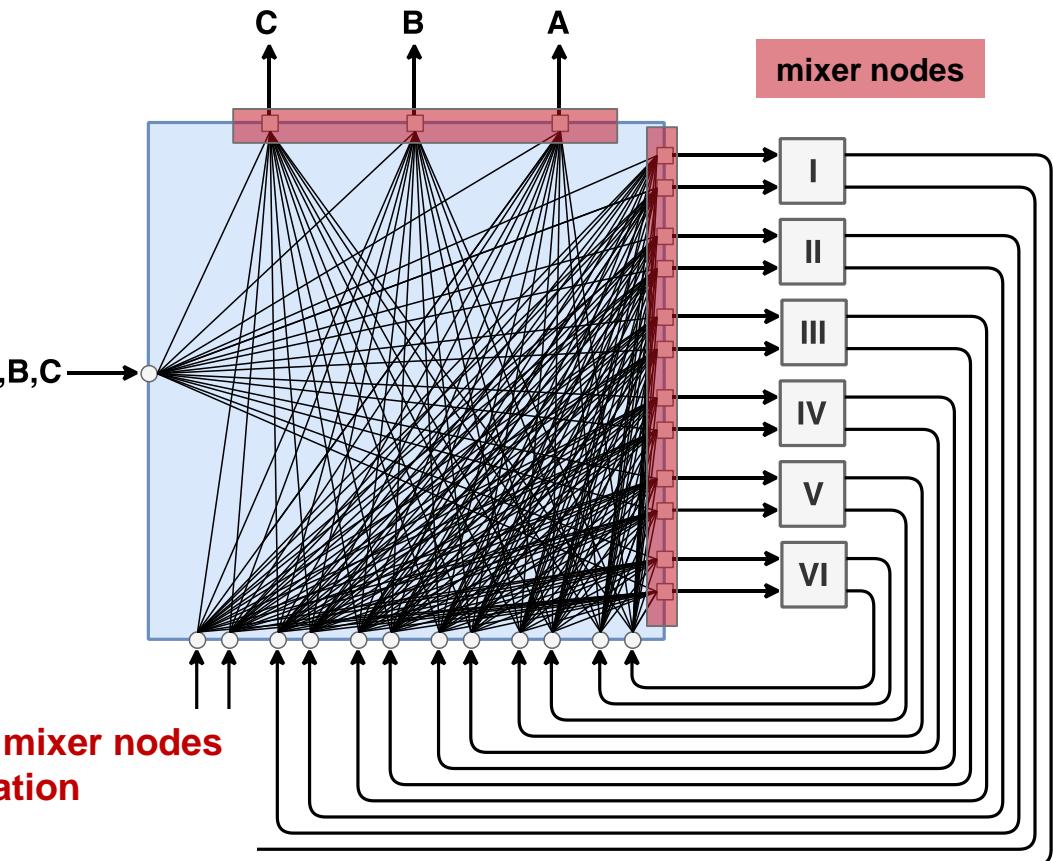
Number of thermo function calls  
per mixer node

property	Calls to thermo package (C++)
single phase enthalpies ( $h^L$ and $h^V$ )	8
distribution coefficients $K_c$	18

→ 26 calls/mixer node → 390 calls/all mixer nodes

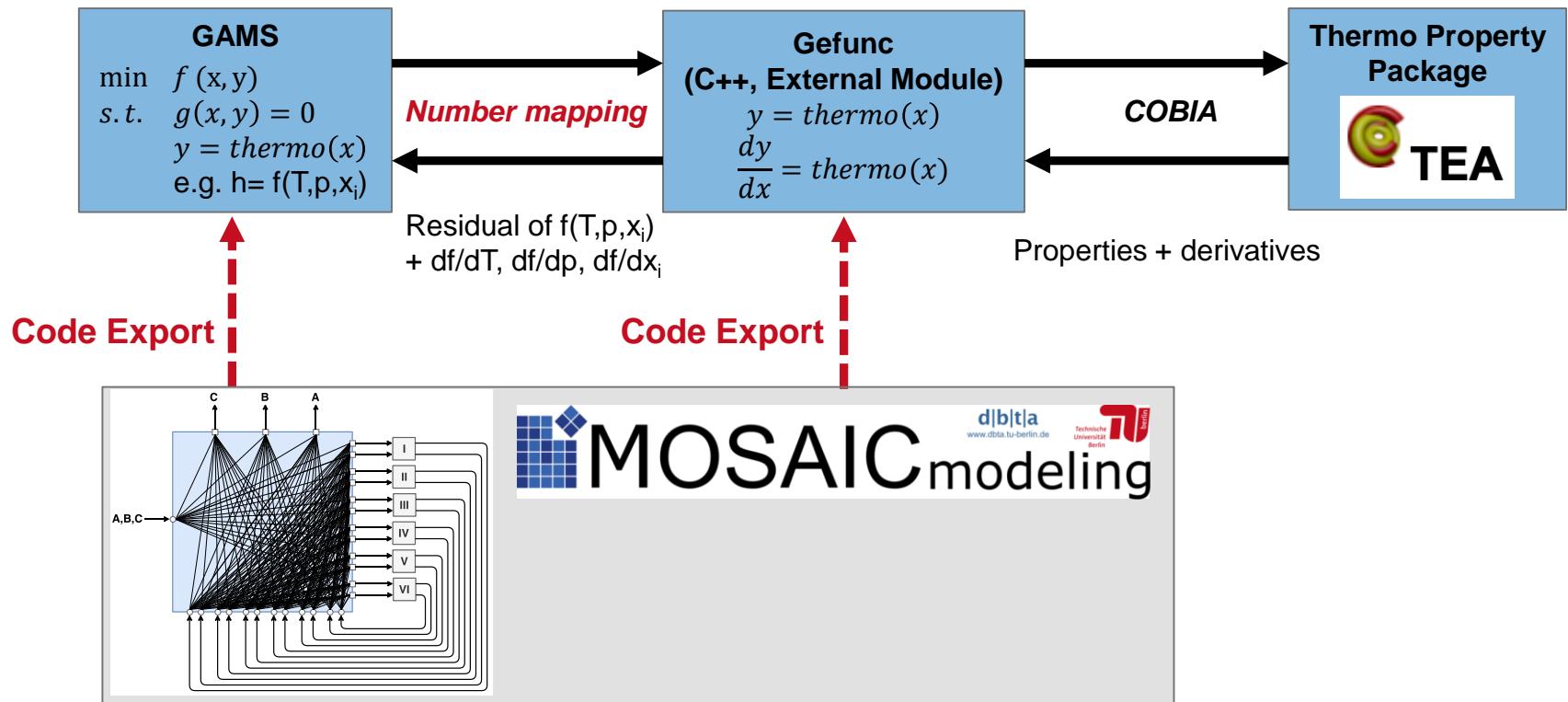
→ 1014 thermo function calls per iteration

Calculation speed is vital!



- 6 Mixed-Integer Optimization in GAMS using CO Thermodynamics via MOSAICmodeling and COBIA for Conceptual Design

## II. Thermo function interface

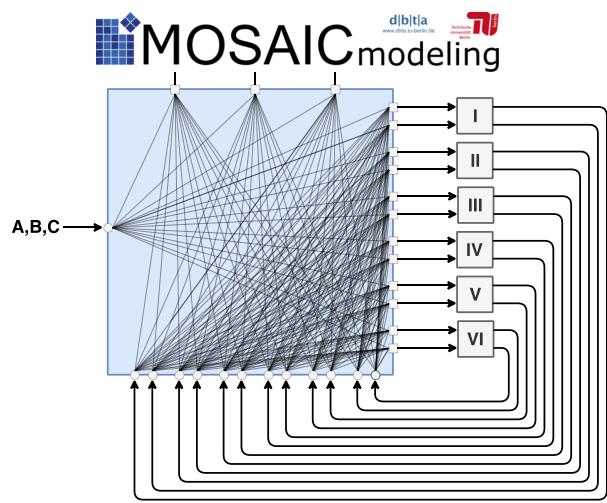


- 7 Mixed-Integer Optimization in GAMS using CO Thermodynamics via MOSAiCmodeling and COBIA for Conceptual Design

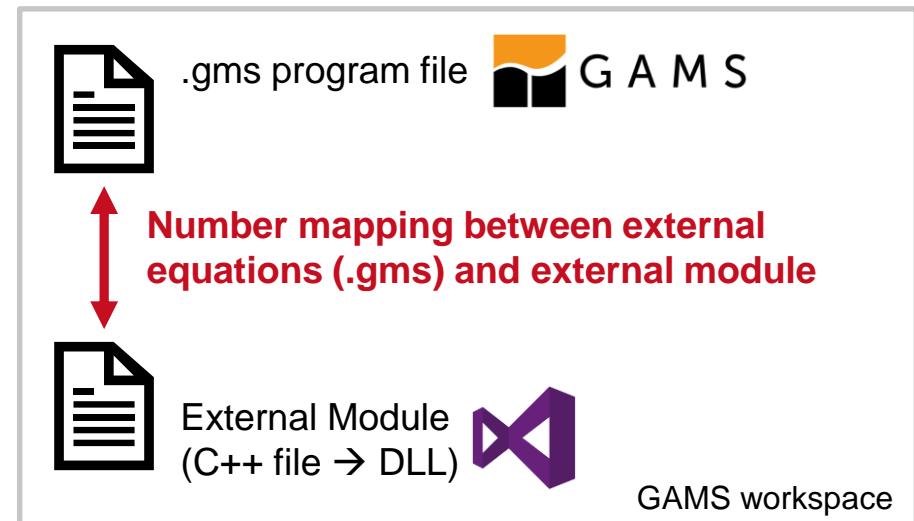
D. Krone | CAPE-OPEN 2019 Annual Meeting | 22.10.2019

## III. Optimization Workflow Overview

### 1. Model development



### 2. Code export



Thermo property package (manager)



3. Thermo calls via COBIA

- 8 Mixed-Integer Optimization in GAMS using CO Thermodynamics via MOSAICmodeling and COBIA for Conceptual Design

D. Krone | CAPE-OPEN 2019 Annual Meeting | 22.10.2019

## III. Optimization Workflow

### 1. Model Development – Equation System

Equation System   Notation

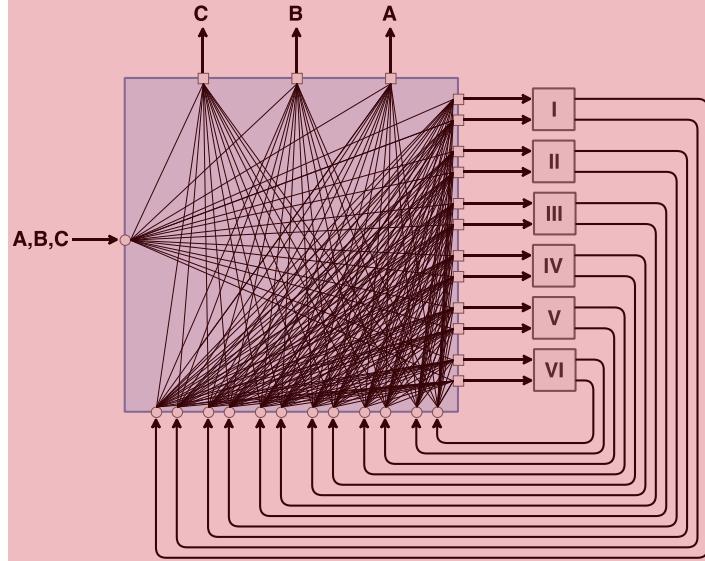
Hierarchy   Equations   Functions   Flowsheet

**All Subsystems of the Superstructure**

Connected Elements

- (1)  $y_{VL,i,j} = \beta_i \cdot K_{VL,i,j}^{VLE} \cdot x_{VL,i,j}$
- (2)  $F_{VL,i-1}^L \cdot h_{VL,i-1}^L + Q_{VL,i}^{\text{trans}} = F_{VL,i}^L \cdot h_{VL,i}^L$
- (3)  $F_{VL,i-1}^L \cdot x_{VL,i-1,j} + F_{VL,i,j}^{\text{trans}} = F_{VL,i}^L \cdot x_{VL,i,j}$
- (4)  $F_{VL,i+1}^V \cdot h_{VL,i+1}^V - Q_{VL,i}^{\text{trans}} = F_{VL,i}^V \cdot h_{VL,i}^V$
- (5)  $F_{VL,i+1}^V \cdot y_{VL,i+1,j} - F_{VL,i,j}^{\text{trans}} = F_{VL,i}^V \cdot y_{VL,i,j}$
- (6)  $\sum_{j=1}^{nc} x_{VL,i,j} = 1$
- (7)  $\sum_{j=1}^{nc} y_{VL,i,j} = 1$
- (8)  $p_{VL}^{\text{PBB}} = p_{VL}$
- (9)  $\beta_i - 1 = s_i^V - s_i^L$
- (10)  $(s_i^L + t_{\text{tol},i}) \cdot (F_{VL,i}^L + t_{\text{tol},i}) - (t_{\text{tol},i})^{2.0} \geq 0$

**System equations**  
**all equations defined generically!**  
 → instantiated before initialization



- 9 Mixed-Integer Optimization in GAMS using CO Thermodynamics via MOSAICmodeling and COBIA for Conceptual Design

## III. Optimization Workflow

### 1. Model Development – Thermo Functions

Function

Location: 112801: fun\_KValues.mosfun

$$K_{VL}^{VLE}(T_{VL}, p_{VL}, x_{VL,j}) = ?$$

Description: empty function for distribution coefficients (vapor-liquid equilibration)

No. of usages: 6

Location: 111441: fun\_enthalpy\_vapor.mosfun

$$h^v(P, T, y_j) = ?$$

Description: function enthalpy vapor

No. of usages: 1

Location: 120080: fun\_enthalpy\_vapor\_with\_composition&minisular\_pre

v

Equation System   Notation

Hierarchy   Equations   Functions   Flowsheet

Function Applications		Input Variables		
Output Variable		Generic	Namespace	Applied
$K_{VL}^{VLE}$	$K_{VL,i,j}^{VLE}$	$p_{VL}$	e0e0e2	$p_{VL}$
		$T_{VL}$	e0e0e2	$T_{VL,i}$
		$x_{VL,j}$	e0e0e2	$x_{VL,i,j}$

Function Applications		Input Variables		
Output Variable		Generic	Namespace	Applied
$K_{VL}^{VLE}$	$K_{VL,i,j}^{VLE}$	$p_{VL}$	e0e1e2	$p_{VL}$
		$T_{VL}$	e0e1e2	$T_{VL,i}$
		$x_{VL,j}$	e0e1e2	$x_{VL,i,j}$

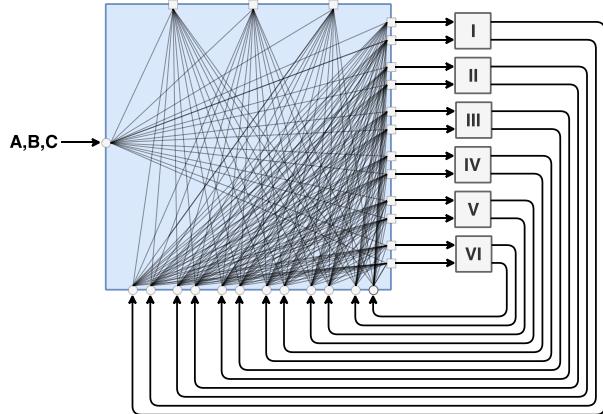
## III. Optimization Workflow

### 2. Code Export

#### 1. Model development



 MOSAIC modeling



#### 2. Code export



.gms program file



G A M S

Number mapping between external equations (.gms) and external module



External Module  
(C++ file → DLL)



GAMS workspace

Thermo property  
package (manager)

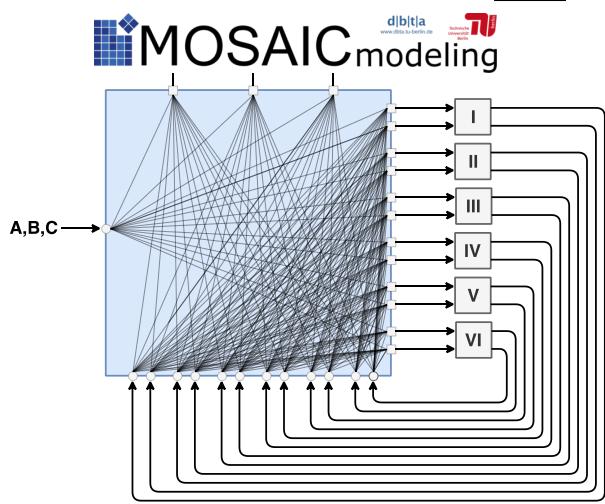


3. Thermo calls via  
COBIA

## III. Optimization Workflow

### 2. Code Export

#### 1. Model development



#### 2. Code export



.gms program file



G A M S

Number mapping between external equations (.gms) and external module



External Module  
(C++ file → DLL)



GAMS workspace

Thermo property package (manager)



3. Thermo calls via COBIA

# III. Optimization Workflow

## Mapping of external equations and external module

```

1 =x= 1*e0e6_h_outL_mix_k1 + 43*e0e6_x_outL_mix_k1_j1 + 44*e0e6_x_outL_mix_k1_j2 + 45*e0e6_x_outL_mix_k1_j3 + 31*e0s0_T + 37*e0s0_p;
2 =x= 2*e0e6_h_outL_mix_k2 + 46*e0e6_x_outL_mix_k2_j1 + 47*e0e6_x_outL_mix_k2_j2 + 48*e0e6_x_outL_mix_k2_j3 + 32*e0s1_T + 38*e0s1_p;
3 =x= 3*e0e6_h_outL_mix_k3 + 49*e0e6_x_outL_mix_k3_j1 + 50*e0e6_x_outL_mix_k3_j2 + 51*e0e6_x_outL_mix_k3_j3 + 33*e0s2_T + 39*e0s2_p;
4 =x= 4*e0e6_h_outL_mix_k4 + 52*e0e6_x_outL_mix_k4_j1 + 53*e0e6_x_outL_mix_k4_j2 + 54*e0e6_x_outL_mix_k4_j3 + 34*e0s3_T + 40*e0s3_p;
5 =x= 5*e0e6_h_outL_mix_k5 + 55*e0e6_x_outL_mix_k5_j1 + 56*e0e6_x_outL_mix_k5_j2 + 57*e0e6_x_outL_mix_k5_j3 + 35*e0s4_T + 41*e0s4_p;;
6 =x= 6*e0e6_h_outL_mix_k6 + 58*e0e6_x_outL_mix_k6_j1 + 59*e0e6_x_outL_mix_k6_j2 + 60*e0e6_x_outL_mix_k6_j3 + 36*e0s5_T + 42*e0s5_p;;
7 =x= 7*e0e6_h_outV_mix_k1 + 61*e0e6_x_outV_mix_k1_j1 + 62*e0e6_x_outV_mix_k1_j2 + 63*e0e6_x_outV_mix_k1_j3 + 37*e0s6_T + 43*e0s6_p;;
8 =x= 8*e0e6_h_outV_mix_k2 + 64*e0e6_x_outV_mix_k2_j1 + 65*e0e6_x_outV_mix_k2_j2 + 66*e0e6_x_outV_mix_k2_j3 + 38*e0s7_T + 44*e0s7_p;;

```

GAMS .gms program file

Referencing by numbers, e.g. for:  $h^L = f(p, T, X_1, X_2, X_3)$

```

if (icntr[I_Eqno] == 1) { //for the first function
    std::vector<double> composition_liquid = { x[42], x[43], x[44] };
    //External Function for liquid enthalpy calculation
    mylocalThermoObj.setSinglePhasePropResult(enthalpyF, x[30], x[36] * 100000.0, liquid, composition_liquid);
    f[0] = (x[0] - mylocalThermoObj.getSinglePhasePropResult(0));
    //Derivatives
    //dh/dh
    d[0] = 1.0;
    //dh/dy_j
    mylocalThermoObj.setSinglePhasePropResult(enthalpyF_DmolFraction, x[30], x[36] * 100000.0, liquid, composition_liquid);
    d[42] = -mylocalThermoObj.getSinglePhasePropResult(0);
    d[43] = -mylocalThermoObj.getSinglePhasePropResult(1);
    d[44] = -mylocalThermoObj.getSinglePhasePropResult(2);
    //dh/dp
    mylocalThermoObj.setSinglePhasePropResult(enthalpyF_Dpressure, x[30], x[36] * 100000.0, liquid, composition_liquid);
    d[36] = -mylocalThermoObj.getSinglePhasePropResult(0)*100000.0; // [1/bar] *
    //dh/dT
    mylocalThermoObj.setSinglePhasePropResult(enthalpyF_Dtemperature, x[30], x[36] * 100000.0, liquid, composition_liquid);
    d[30] = -mylocalThermoObj.getSinglePhasePropResult(0); // [1/K]
}

```

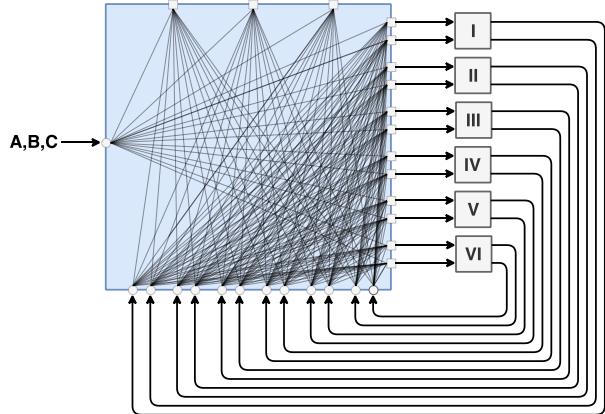
External Module (C++ file → DLL) 

### III. Optimization Workflow

#### 3. Thermo calls to Thermo package via COBIA

##### 1. Model development

 MOSAIC modeling



##### 2. Code export



.gms program file



G A M S

Number mapping between external equations (.gms) and external module



External Module  
(C++ file → DLL)



GAMS workspace

Thermo property  
package (manager)

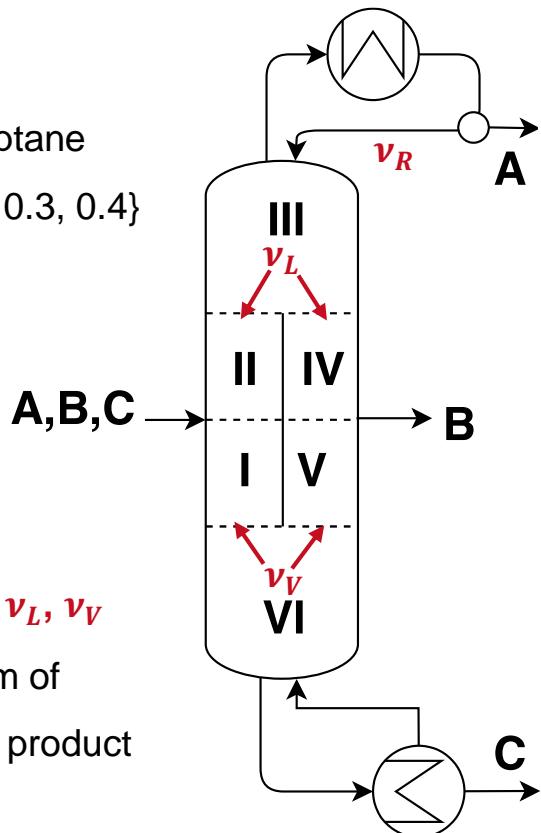


3. Thermo calls via  
COBIA 

## IV. Case Studies – Separation of ternary mixture via a dividing wall column (DWC)

- **Feed specifications:**

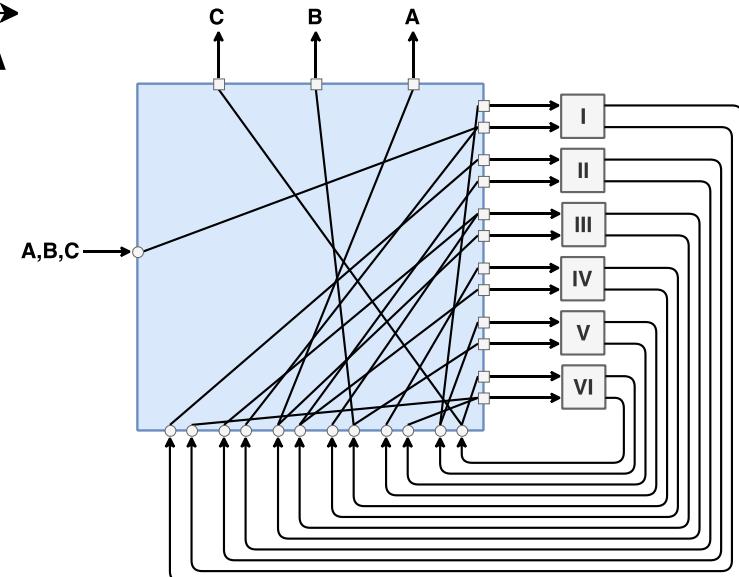
- n-pentane/n-hexane/n-heptane
- Molar composition = {0.3, 0.3, 0.4}
- T = 325 K
- p = 1 bar
- F = 1 kmol/s



- **Optimization task:**

find optimal split ratios:  $v_R, v_L, v_V$

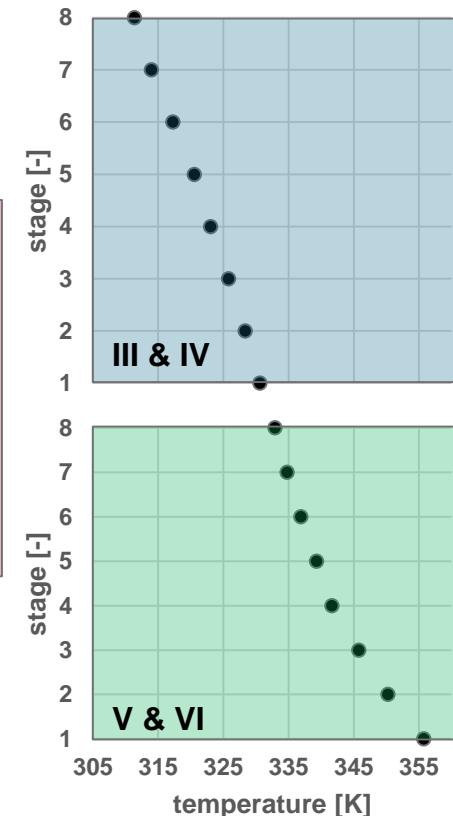
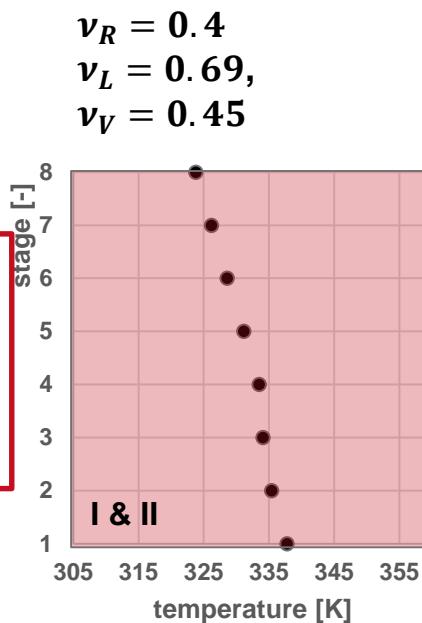
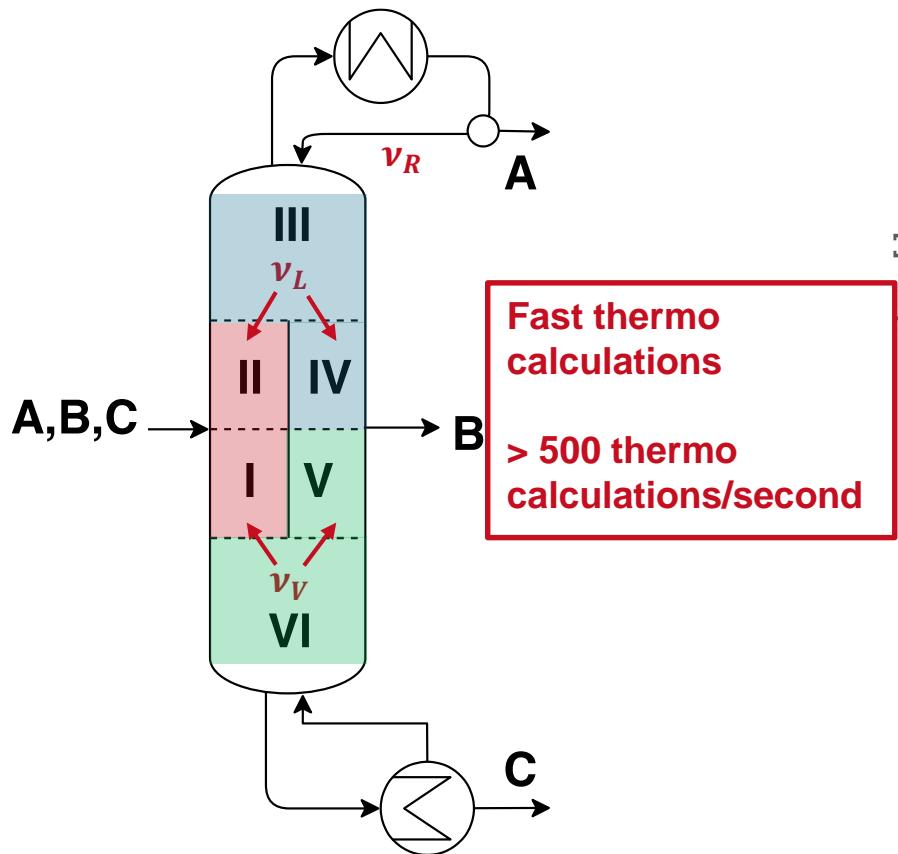
- **Maximizing objective:** sum of desired product qualities in product streams



**Only showing active superstructure connections**

## IV. Case Studies – Separation of ternary mixture

### Speed of thermo calculations and profiles



## V. Summary

**Functioning interface between  
GAMS and CAPE-OPEN thermo property packages**

**For fast thermo calculations**

**Reducing modelling effort by automatic code generation  
with MOSAICmodeling**

- 17 Mixed-Integer Optimization in GAMS using CO Thermodynamics via MOSAICmodeling and COBIA for Conceptual Design

D. Krone | CAPE-OPEN 2019 Annual Meeting | 22.10.2019

## Acknowledgements

**Jasper van Baten**  
for his code examples and help with COBIA

- 18 Mixed-Integer Optimization in GAMS using CO Thermodynamics via MOSAIComodeling and COBIA for Conceptual Design

D. Krone | CAPE-OPEN 2019 Annual Meeting | 22.10.2019

## Questions and answers



## II. Results – separation of ternary mixture

### ii. Optimization of the superstructure in DWC configuration

#### Product flows and qualities:

##### Destillate:

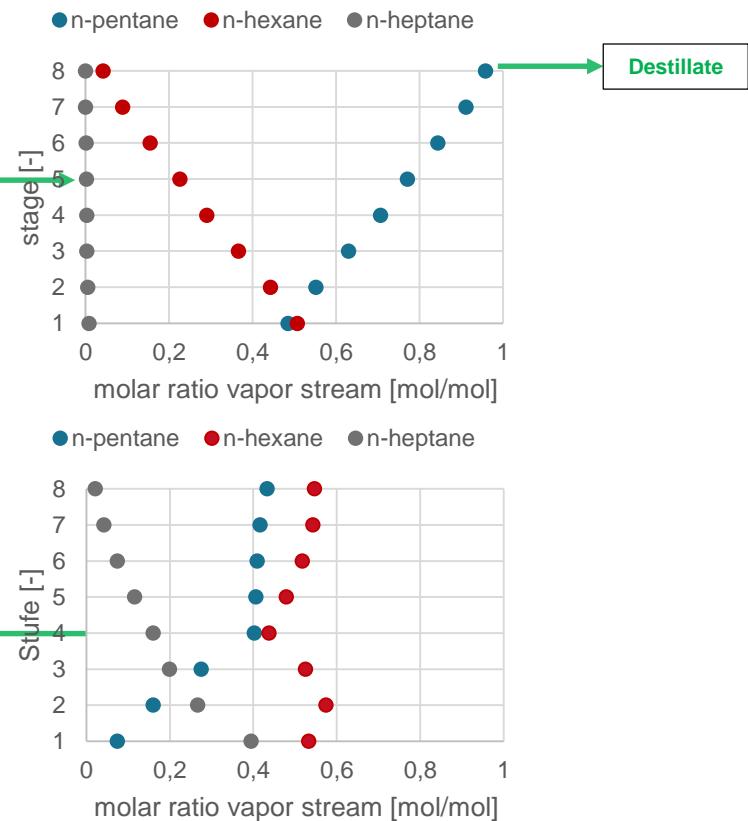
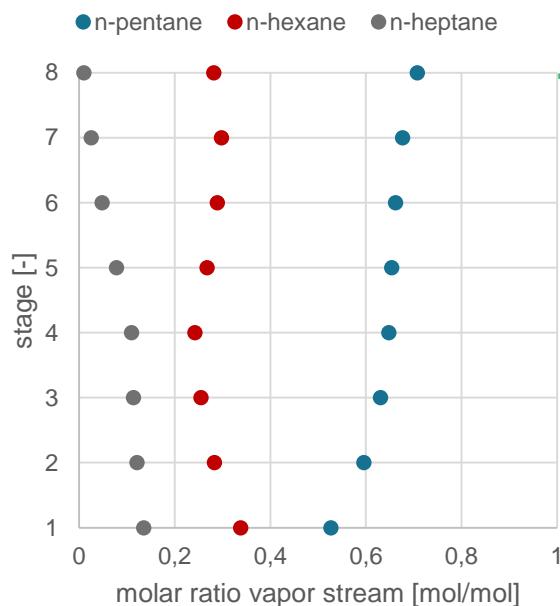
- $F = 0,2752 \text{ kmol/s}$
- Composition =  $\{0.958, 0.042, 0.000\}$

##### Middle draw:

- $F = 0,1 \text{ kmol/s}$
- Composition =  $\{0.241, 0.726, 0.032\}$

##### Bottom draw:

- $F = 0,6236 \text{ kmol/s}$
- Composition =  $\{0.019, 0.345, 0.636\}$



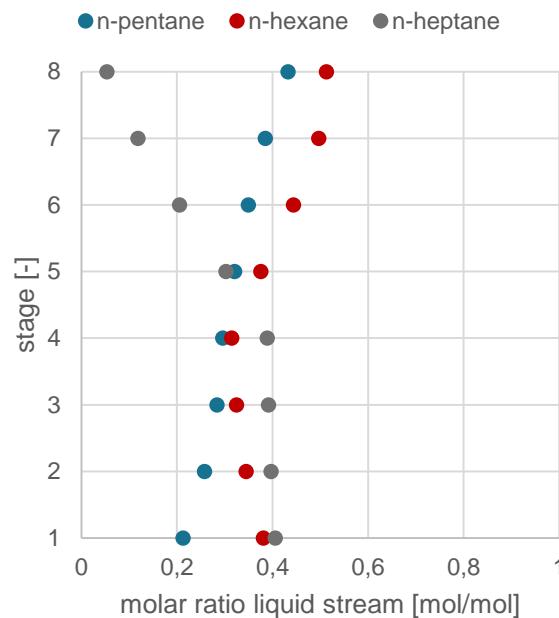
## II. Results – separation of ternary mixture

### ii. Optimization of the superstructure in DWC configuration

#### Product flows and qualities:

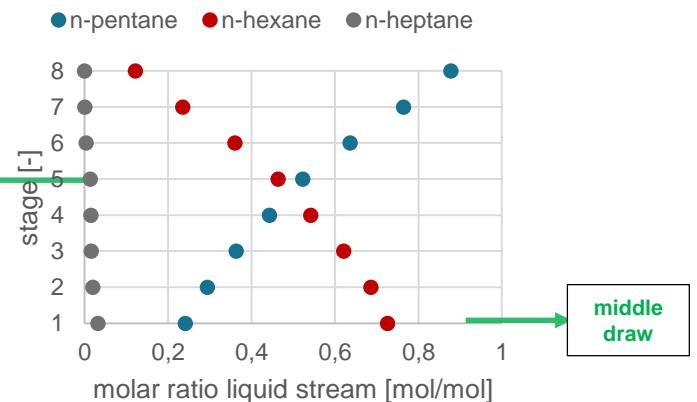
##### Destillate:

- $F = 0,2752 \text{ kmol/s}$
- Composition =  
 $\{0.958, 0.042, 0.000\}$



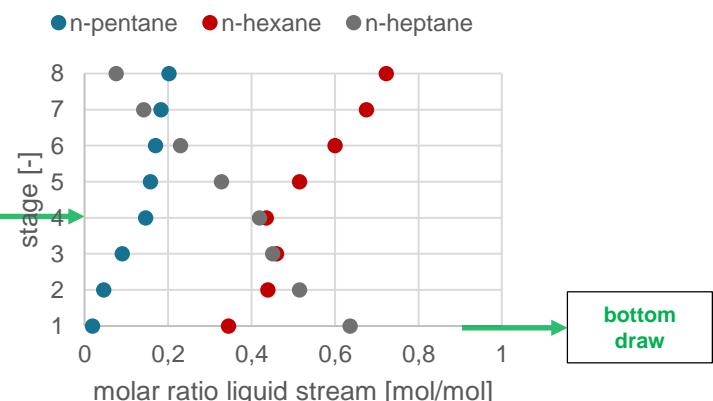
##### Middle draw:

- $F = 0,1 \text{ kmol/s}$
- Composition =  
 $\{0.241, 0.726, 0.032\}$



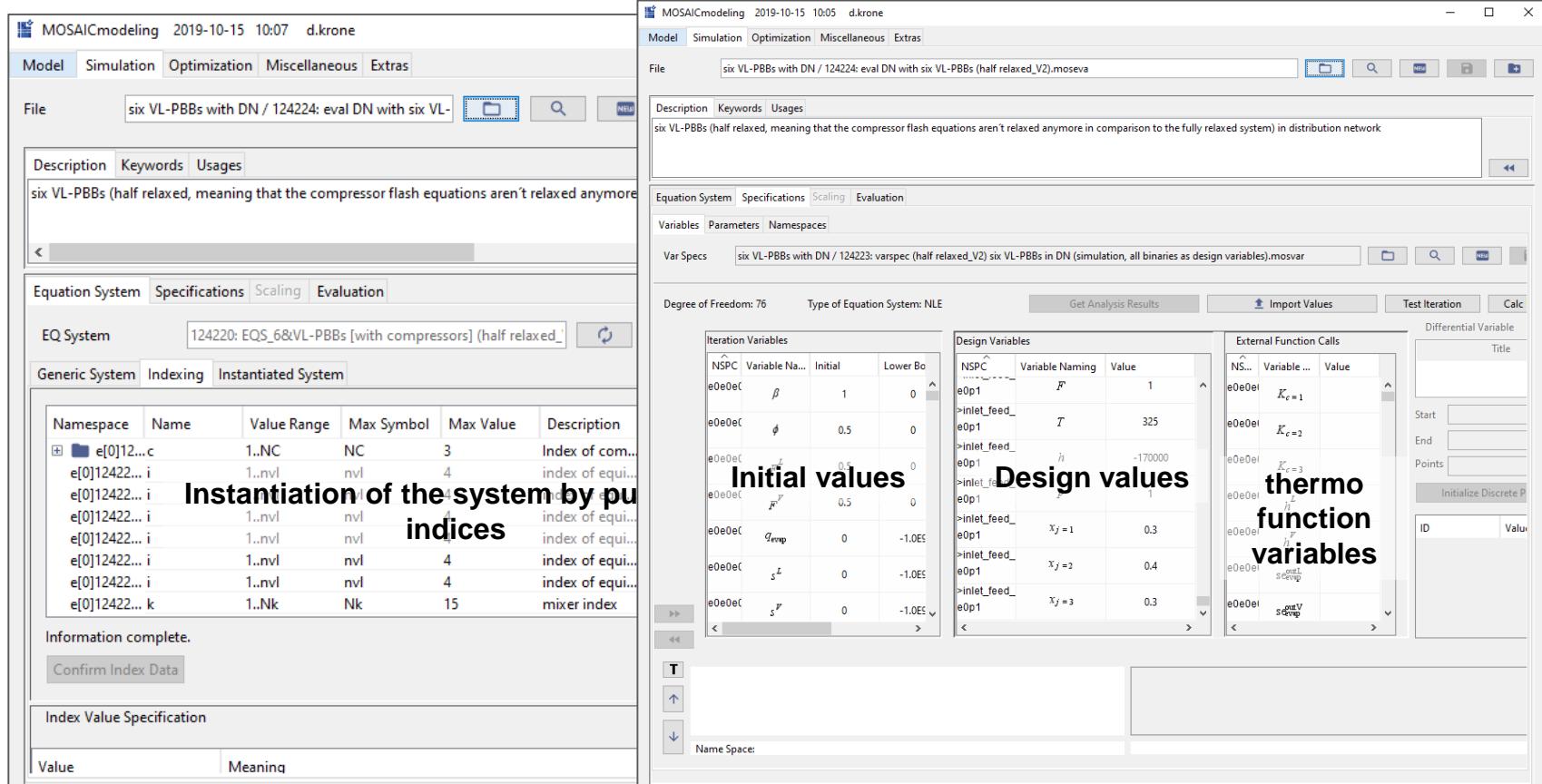
##### Bottom draw:

- $F = 0,6236 \text{ kmol/s}$
- Composition =  
 $\{0.019, 0.345, 0.636\}$



## II. Optimization Workflow

### 1. Model Development – Instantiation and Initialisation



The screenshot shows two windows of the MOSAICmodeling software.

**Left Window (Instantiation):**

- Title Bar:** MOSAICmodeling 2019-10-15 10:07 d.krone
- Menu Bar:** Model, Simulation, Optimization, Miscellaneous, Extras
- File Bar:** six VL-PBBs with DN / 124224: eval DN with six VL-PBBs with DN / 124224: eval DN with six VL-PBBs (half relaxed\_V2).moseva
- Description Tab:** six VL-PBBs (half relaxed, meaning that the compressor flash equations aren't relaxed anymore)
- Equation System Tab:** EQ System 124220: EQS\_6&VL-PBBs [with compressors] (half relaxed\_)
- Table:** Shows namespace, name, value range, max symbol, max value, and description for various variables. One row is highlighted with a red box and labeled "Instantiation of the system by pu indices".
- Information:** Information complete. Confirm Index Data button.

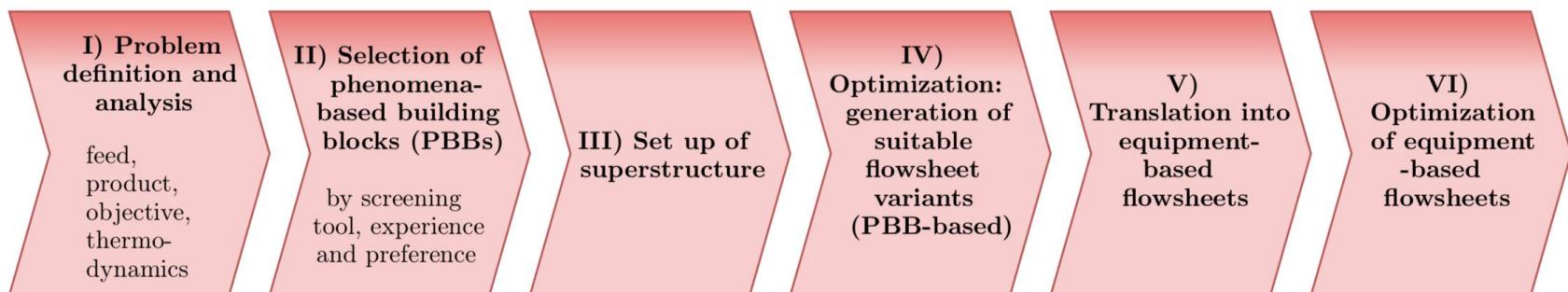
**Right Window (Initialisation):**

- Title Bar:** MOSAICmodeling 2019-10-15 10:05 d.krone
- Menu Bar:** Model, Simulation, Optimization, Miscellaneous, Extras
- File Bar:** six VL-PBBs with DN / 124224: eval DN with six VL-PBBs (half relaxed\_V2).moseva
- Description Tab:** six VL-PBBs (half relaxed, meaning that the compressor flash equations aren't relaxed anymore in comparison to the fully relaxed system) in distribution network
- Equation System Tab:** Var Specs six VL-PBBs with DN / 124223: varspec (half relaxed\_V2) six VL-PBBs in DN (simulation, all binaries as design variables),mosvar
- Tables:**
  - Iteration Variables:** Shows variables like  $\beta$ ,  $\phi$ ,  $F$ ,  $T$ ,  $j$ ,  $q_{\text{exp}}$ ,  $s^L$ ,  $s^R$ .
  - Design Variables:** Shows variables like  $F$ ,  $T$ ,  $j$ ,  $x_j = 1, 2, 3$ .
  - External Function Calls:** Shows function calls like  $K_c = 1, 2$ ,  $K_{c,2}$ ,  $s_{\text{exp}}$ ,  $s_{\text{exp}}^V$ .
- Text Labels:** "Initial values", "Design values", "thermo function variables".
- Buttons:** Get Analysis Results, Import Values, Test Iteration, Calc.

## 22 Mixed-Integer Optimization in GAMS using CO Thermodynamics via MOSAICmodeling and COBIA for Conceptual Design

D. Krone | CAPE-OPEN 2019 Annual Meeting | 22.10.2019

# I. Process synthesis approach



**Fig. 1: Process synthesis approach based on the framework by Kuhlmann and Skiborowski<sup>1</sup>**

<sup>1</sup> H. Kuhlmann, M. Skiborowski, Ind. Eng. Chem. Res. 2017, 56 (45), 13461 - 13481

23 Mixed-Integer Optimization in GAMS using CO Thermodynamics via MOSAIChModeling and COBIA for Conceptual Design

D. Krone | CAPE-OPEN 2019 Annual Meeting | 22.10.2019